New limits on the mass of neutral Higgses in General Models

D. Comelli[*]

Instituto de Física Corpuscular - IFIC/CSIC

Dept. de Física Teòrica, Universidad de València

46100 Burjassot, València. Spain

and

J.R. Espinosa [†]

Deutsches Elektronen Synchrotron DESY.

Notkestrasse 85. 22603 Hamburg. Germany

Abstract

In general electroweak models with weakly coupled (and otherwise arbitrary) Higgs sector there always exists in the spectrum a scalar state with mass controlled by the electroweak scale. A new and simple recipe to compute an analytical tree-level upper bound on the mass of this light scalar is given. We compare this new bound with similar ones existing in the literature and show how to extract extra information on heavier neutral scalars in the spectrum from the interplay of independent bounds. Production of these states at future colliders is addressed and the implications for the decoupling limit in which only one Higgs is expected to remain light are discussed.

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In the Standard Model the mass of the Higgs boson is an unknown parameter waiting to be measured and all the information we can provide about it to guide its search is precious. Lower bounds on it can be obtained requiring the electroweak minimum of the Higgs potential to be stable or metastable and upper bounds can also be found imposing that the Standard Model remains perturbative up to some high energy scale Λ. This last point is made possible by the fact that the squared mass m_h^2 of the Higgs boson is of the form λv^2 with v fixed by the gauge boson masses and λ being the (not asymptotically free) scalar self coupling. Imposing that λ remains perturbative below Λ one gets an upper bound on m_h, especifically $m_h < 180 - 200 \ GeV$ for $\Lambda = 10^{16} \ GeV$.

In more general models of electroweak breaking one typically has more physical Higgs bosons (scalars, pseudoscalars, charged,...). The Higgs potential will contain more mass parameters and the mass spectrum of the Higgs sector will be more complicated. However, in any model of electroweak breaking with weakly coupled Higgs sector at least one of the physical Higgs bosons has a mass controlled by the electroweak scale [1, 2], i.e. there is an upper bound on the squared mass of the lightest Higgs boson of the form λv^2, with λ some combination of quartic Higgs self couplings and v of the order of the electroweak scale. This means that in the most general model one can always find an upper bound on (at least) one Higgs scalar imposing that the theory remains perturbative up to some high energy scale.

In section 1 we present a novel proof of this fact deriving a very simple mass bound, different and independent of those presented in [1, 2]. This bound can be applied in many contexts and models. As a relevant example we derive in section 2 the form of this bound in general supersymmetric Standard models. The new bound is compared with the old ones of refs. [1, 2] in section 3 while section 4 is devoted to extract some implications of the interplay between different bounds. Finally, section 5 examines the production cross section of these light neutral scalars in general models.

1. Upper limit on the mass of the lightest neutral scalar

The proof goes as follows: let Φ_j be the (generically complex) neutral scalar fields in the model (i.e. all those fields susceptible of taking a VEV). They belong to $(2T_j + 1) - SU(2)_L$ multiplets and have hypercharges Y_j. We write:

$$\Phi_j = \frac{1}{\sqrt{2}} \left[\phi_j^{0r} + i\phi_j^{0i} \right]. \tag{1}$$

After the breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$ we define the real field ϕ^0 as:

$$\phi^0 = \frac{1}{\langle \phi^0 \rangle} {\sum_j}' \left[\langle \phi_j^{0r} \rangle \phi_j^{0r} + \langle \phi_j^{0i} \rangle \phi_j^{0i} \right], \tag{2}$$

with

$$\langle \phi^0 \rangle^2 = 2 {\sum_j}' |\langle \Phi_j \rangle|^2, \tag{3}$$

and the primed sum extends only to $2T_j$ odd fields (i.e. doublets, 4-plets, etc. but not singlets, triplets,...). The reason for this will be clear later on. Definition (2) ensures that
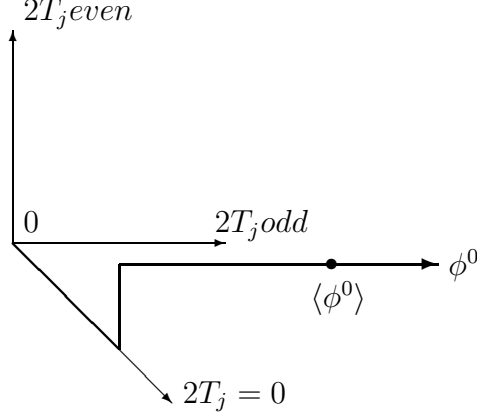
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Figure 1: The structure of electroweak breaking

ϕ^0 will be the only $2T_j$ odd field having a non-zero VEV. All other $2T_j$ odd fields orthogonal to ϕ^0 have zero VEV by construction.

The structure of the electroweak breaking is represented pictorially in Fig. 1. The breaking can be decomposed in $2T_j$ even and odd parts. The $2T_j = 0$ VEV (coming from singlets) has been drawn separately because it does not break $SU(2) \times U(1)$. We can calculate the tensor M_{ij}^2 of (scalar neutral) mass excitations in the vacuum $\langle \phi^0 \rangle$ and in particular we will be interested in excitations along the ϕ^0-direction: call the corresponding squared mass $M_{\phi^0}^2 \equiv \langle \phi^0 | M^2 | \phi^0 \rangle$. This mass provides an upper bound on the mass of the lightest neutral scalar of the theory: if $M_{\phi^0}^2$ is a true eigenvalue of M^2 then the bound is saturated. If it is not a true eigenvalue then $\langle \phi^0 | M^2 | \phi^0 \rangle$ is not stable under small perturbations in the field direction ϕ^0, that is, we can find some field direction along which this mass is reduced and so, some eigenvalue of M^2 lies below $M_{\phi^0}^2$. (And that eigenvalue is not the neutral Goldstone: note that $M_{\phi^0}^2$ will be stable under perturbations along the "Goldstone direction").

Having proved that the quantity $M_{\phi^0}^2$ is a bound on the mass of the lightest neutral scalar in the theory we now calculate its general form. For this we need to consider the scalar potential along the direction ϕ^0. Towards this end, consider the (multiplicative) discrete symmetry

$$\Phi_j \to (-1)^{2T_j} \Phi_j, \tag{4}$$

(that can be extended to all the fields in the theory). The Lagrangian being a $SU(2)_L$-singlet is invariant under (4). When the $2T_j$-even fields take a VEV, the symmetry remains unbroken. So, $V(\phi^0)$ must respect also this symmetry. As $\phi^0 \to -\phi^0$ under the transformation (4), the form of $V(\phi^0)$ is restricted to be, at tree level:

$$V(\phi^0) = V_0 - \frac{1}{2}m^2 \phi^0 \phi^0 + \frac{1}{8}\lambda (\phi^0 \phi^0)^2, \tag{5}$$

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with cubic and linear terms forbidden. To see how this comes out in detail consider a linear term $\delta V = \kappa \phi^0$ in the potential. If this comes from a full scalar potential $SU(2)_L$ invariant, then κ should be the VEV of a field or combination of fields transforming with $2T$ odd. The first case ($\kappa \sim \langle \phi_j \rangle$) is not possible because all $2T_i$ odd fields orthogonal to ϕ^0 have zero VEV. The second case ($\kappa \sim \langle \phi_i \phi_j ... \rangle$) is also impossible because some of the fields ϕ_i appearing in κ must transform as $2T_i$ odd if κ itself is $2T$ odd (the combination of $2T_i$ even fields produces only $2T$ even fields), and again those have been already projected into ϕ^0. A similar argument applies to the cubic terms in $V(\phi^0)$.

Then, from eq. (5), using the condition $\partial V / \partial \phi^0 = 0$ at the minimum $\langle \phi^0 \rangle = v$ we get

$$M_{\phi^0}^2 = \lambda v^2, \qquad (6)$$

where v is related to the electroweak scale through the gauge boson masses [it will be different from model to model but always $v^2 \leq (\sqrt{2} G_F)^{-1}$], and λ is some combination of quartic couplings of the theory so that it is sensible to require it to be perturbative.

We have proved our $SU(2)$-based theorem extracting from $SU(2)_L$ a discrete symmetry to restrict the form of the potential in some field direction. We can follow the same procedure for any spontaneously broken $U(1)$ (global or local). Our $U(1)$ derivation differs from a similar one given in [3], the conclusion being the same. Let us concentrate in the neutral fields ϕ_i which have a non-zero $U(1)$ charge Q_i (the Q_i's are some fractional numbers, $Q_i = n_i / d_i$) and take a VEV. We can always rescale the charges multiplying them by L, the least common multiple of the denominators d_i in such a way that the new charges $Q_i' = L Q_i$ are integers. The potential for the fields ϕ_i will be invariant under the discrete symmetry

$$\phi_i \rightarrow (-1)^{Q_i'} \phi_i. \qquad (7)$$

We can decompose the VEVs in a singlet, an odd and an even part according to its properties under (7). Then, a mass bound can be derived looking in the field direction determined by

$$\phi^0 = \frac{1}{\langle \phi^0 \rangle} \sum_j {}' \left[\langle \phi_j^{0r} \rangle \phi_j^{0r} + \langle \phi_j^{0i} \rangle \phi_j^{0i} \right], \qquad (8)$$

where the primed sum extends to odd Q_i' fields only.

We can apply this result to the breaking of $U(1)_Y$. As we are always assuming that only neutral fields take a VEV we have the relation $Y_i = -T_{3i}$ for the i-th field. The T_{3i} are integers or half integers. If some half-integer Y_i field is taking a VEV [e.g. some $SU(2)_L$ doublet, as required to give masses to quarks] then L=2 and the discrete symmetry (7) is

$$\phi_i \rightarrow (-1)^{L Y_i} \phi_i = (-1)^{2 T_{3i}} \phi_i = (-1)^{2 T_i} \phi_i, \qquad (9)$$

and this is nothing but the old $SU(2)$-based discrete symmetry (4) so that no further information can be extracted from the $U(1)_Y$ breaking[1].

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[1]In case that only integer Y_i fields develop a VEV, $L = 1$ and now the mass bound can be derived from the $U(1)_Y$ breaking while the $SU(2)_L$ breaking gives no information.

2. Supersymmetric models

In the Minimal Supersymmetric Standard Model the Higgs sector contains two Higgs doublets (H_1, H_2) so that, after electroweak symmetry breaking $(\langle H_1^0 \rangle = v_1/\sqrt{2}, \langle H_2^0 \rangle = v_2/\sqrt{2})$ the spectrum of physical Higgses consists of two scalars, h^0 and H^0, one pseudoscalar, A^0, and a pair of charged Higgses $H^\pm$. On the other hand supersymmetry and gauge invariance restrict the interactions in the Higgs sector in such a way that the mass spectrum is quite constrained and at tree level it is completely determined by just two parameters: $\tan \beta = v_2/v_1 = \langle H_2^0 \rangle / \langle H_1^0 \rangle$ and the mass of one of the Higgses, conventionally taken to be m_{A^0}.

As is well known, in the Minimal Supersymmetric Standard Model the tree level mass m_h of the lightest scalar h^0 is bounded by $M_Z |\cos 2\beta|$. Radiative corrections to the mass of this Higgs boson can be large if the mass of top and stops is large, and the tree level bound can be spoiled. But, even if the lightest Higgs can scape detection at LEP-II its mass is always of the order of the electroweak scale (the dependence on the soft breaking scale is only logarithmic).

The situation in extended supersymmetric models is somewhat qualitatively different. An analytical upper bound on the tree-level mass of the lightest Higgs boson is known for very general Supersymmetric Standard Models with extended Higgs or gauge sectors [4, 5]. This bound depends on the electroweak scale (given by M_Z) and on the new Yukawa or gauge couplings that appear in the theory, which are not fixed by experiment as in the MSSM. Numerical bounds can be obtained by putting limits on these unknown parameters (*e.g.* assuming that the theory remains perturbative up to some high scale). These bounds are typically greater than the MSSM bound but still of order M_Z, for example [4] one gets $m_h < 155 \ GeV$ for $\Lambda = 10^{16} \ GeV$.

After the results of section 1 it should be clear that there is nothing special concerning supersymmetry in what respects to the bound on the lightest Higgs boson: the existence of the mass bound controlled by the electroweak scale follows directly from gauge invariance without the need to advocate supersymmetry [the role of supersymmetry is to fix in some cases (*e.g.* the MSSM) the quartic couplings appearing in the bound].

Following section 1, the derivation of the tree-level upper bound on the mass of the lightest Higgs boson is greatly simplified: Consider the most general supersymmetric standard model assuming for simplicity that CP is conserved and all $\phi_j^{0i} = 0$. Define the field Φ^0 according to (2) and calculate the quartic term $(\phi^0)^4$ in the potential. This comes from two contributions:

i) F-Terms: write the superpotential for the neutral fields in terms of Φ^0 plus orthogonal fields. The only terms that will contribute to $(\phi^0)^4$ in the potential are of the form

$$\delta f = \sum_{ijk} \lambda_{ijk} \varphi_i^0 \Phi_j^0 \Phi_k^0 = \sum_{ijk} \lambda_{ijk} c_j c_k \varphi_i^0 \Phi^0 \Phi^0 + ... \equiv \sum_i \lambda_i \varphi_i^0 \Phi^0 \Phi^0 + ..., \qquad (10)$$

where now the fields φ_i are $2T_i$ even and $c_j = \langle \Phi_j^0 \rangle / \langle \Phi^0 \rangle$ appear from the projection of odd-fields onto Φ^0. Then it results

$$\delta V = \frac{1}{4} \sum_i \lambda_i^2 (\phi^0 \phi^0)^2. \qquad (11)$$

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ii) D-terms: The contribution from $SU(2)_L \times U(1)_Y$ is easily calculable and gives

$$\delta V = \frac{1}{8}(g^2 + g'^2) \left({\sum_j}' Y_j c_j^2 \right)^2 (\phi^0 \phi^0)^2, \tag{12}$$

where $c_j = \langle \phi_j^0 \rangle / \langle \phi^0 \rangle$. The contribution from other gauge groups that might be present can also be added without difficulty following the same procedure.

Putting together the contributions coming from (11) and (12) the bound on the mass of the lightest Higgs boson can be written as

$$m_h^2 \leq (g^2 + g'^2) \left({\sum_j}' Y_j c_j^2 \right)^2 v_o^2 + 2 \sum_i \lambda_i^2 v_o^2, \tag{13}$$

where $v_o = \langle \phi^0 \rangle$. The subscript is meant to remind that only odd-fields contribute to v_o.

Previous studies of this kind in general supersymmetric models analyzed 2×2 submatrices of the neutral scalar matrix. We have shown that this is an unnecessary complication and moreover we can see that the naive application of this technique can fail in some cases (see Appendix). In the light of the case presented in the Appendix one can wonder whether the universal upper bound calculated numerically for M_h in models with an arbitrary Higgs sector [4] remains valid or gets modified (after all the technique of 2×2 submatrices was used to extract the analytic bound). The key point here is that, to affect the derivation of the analytical bound obtained in [4] one needs some even (non-singlet) fields with non-zero VEV. In such a case v_o is proportionally reduced with a corresponding decrease in the bound on M_h. For that reason, when looking for the universal upper bound in models with arbitrary Higgs sector one has to concentrate only in those cases with $v_e = 0$, as was done in [4, 6].

3. Comparison with existing bounds

Bounds similar to the one we presented in Section 1 have been previously derived in the literature [1, 2]. All of them have the form λv^2, (λ being some quartic scalar coupling and v some VEV controlled by the measured gauge boson masses) but are generically different as they arise from looking into different field space directions. We review here those bounds in a unified manner that should help the comparison between them.

All the bounds are obtained from an inequality of the form

$$\langle \varphi_a | M^2 | \phi_a \rangle \leq \lambda_a v_a^2, \tag{14}$$

where the index a just runs over different bounds. The states φ_a and ϕ_a are some real fields that can be normalized to satisfy $\langle \varphi_a | \phi_a \rangle = 1$ (and in most cases $\varphi_a = \phi_a$). The final form of the bounds follow from (14) immediately[2]:

$$\begin{aligned} \lambda_a v_a^2 &\geq \langle \varphi_a | M^2 | \phi_a \rangle = \sum_{A,B} \langle \varphi_a | A \rangle M_A^2 \delta_{AB} \langle B | \phi_a \rangle \\ &= \sum_A M_A^2 \langle \varphi_a | A \rangle \langle A | \phi_a \rangle \geq M_h^2 \langle \varphi_a | \phi_a \rangle = M_h^2, \end{aligned} \tag{15}$$

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[2]If $\phi_a \neq \varphi_a$ the condition $\langle \varphi_a | A \rangle \langle A | \phi_a \rangle \geq 0$ must be satisfied.

where the $|A\rangle$'s form a basis of neutral scalar mass eigenstates. The form (14) will be useful for the discussion of the decoupling limit in Section 4. In particular, knowledge of the field directions φ_a and ϕ_a will be needed. We give them below together with the different mass bounds on M_h^2. To simplify we will assume that CP is conserved in the Higgs sector and all the VEVs can taken to be real.

Bound 1. It was obtained in ref. [1] looking at the potential in the direction

$$\varphi_1 \equiv \phi_1 = \frac{1}{v_1} \sum_i \langle h_i^{0r} \rangle h_i^{0r}, \tag{16}$$

with

$$v_1^2 = \sum_i \langle h_i^{0r} \rangle^2 = v_o^2 + v_e^2. \tag{17}$$

Here the sum runs over all non-singlets, even or odd, as indicated by the last equality.

The tree-level potential $V(\phi_1)$ has no linear terms in ϕ_1 but cubic ones are now allowed:

$$V(\phi_1) = V_0 + m^2 \phi_1^2 + \sigma \phi_1^3 + \frac{1}{8} \lambda_1 \phi_1^4. \tag{18}$$

Nevertheless, a bound can be obtained assuming that the electroweak vacuum is the deepest one (the bound would not apply if the vacuum were metastable at tree level) and takes the form

$$M_h^2 \leq \lambda_1 (v_o^2 + v_e^2). \tag{19}$$

Bounds 2,3,4. These three bounds were obtained in ref. [2] making use of gauge invariance to relate different order derivatives of the effective potential. In this way second derivatives (masses) can be connected to fourth derivatives (quartic couplings) and the bounds follow. In fact it is remarkable that they apply to very general non-polynomial potentials although here we will restrict ourselves to tree-level polynomial potentials.

The field directions in (14) are

$$\varphi_2 \equiv \phi_2 \ \sim \ \sum_i t_{3i}^2 \langle h_i^{0r} \rangle h_i^{0r} \sim \varphi_3, \tag{20}$$

$$\phi_3 \ \sim \ \sum_i [t_i(t_i+1) - t_{3i}^2] \langle h_i^{0r} \rangle h_i^{0r} \sim \varphi_4 \equiv \phi_4. \tag{21}$$

The quartic couplings can be read off the quartic potential for the Goldstone bosons

$$\delta V = \frac{1}{24} \lambda_2 G_0^4 + \frac{1}{2} \lambda_3 G_0^2 G^+ G^- + \frac{1}{4} \lambda_4 (G^+ G^-)^2, \tag{22}$$

where

$$G_0 = \frac{g}{M_Z \cos \theta_W} \sum_i t_{3i} \langle h_i^{0r} \rangle h_i^{0i}, \tag{23}$$

and

$$G^+ = \frac{g}{\sqrt{2} M_W} \sum_{ij} [(t_{ij}^+ \langle h_j^{0r} \rangle) \Phi_i - \Phi_i^\dagger (t_{ij}^- \langle h_j^{0r} \rangle)], \tag{24}$$

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with $\Phi_i = (h_i^{0r} + ih_i^{0i})/\sqrt{2}$ as usual.

The final form of the bounds is

$$M_h^2 \ \leq \ \frac{1}{3}\lambda_2(v_o^2 + v_e^2), \tag{25}$$

$$M_h^2 \ \leq \ \lambda_3\rho\frac{1}{\sqrt{2}G_F} \equiv \lambda_3\rho v_W^2, \tag{26}$$

$$M_h^2 \ \leq \ \frac{1}{2}\lambda_4(v_o^2 + v_e^2). \tag{27}$$

Here $\rho = M_W^2/(M_Z^2 \cos\theta_W)$ and $v_W^2 \geq v_o^2 + v_e^2$.

Bound 5. The one we derived in Section 1. It has

$$\varphi_5 \equiv \phi_5 = \frac{1}{v_o}\sum_i{}' \langle h_i^{0r}\rangle h_i^{0r}, \tag{28}$$

with

$$\delta V = \frac{1}{8}\lambda_5\phi_5^4, \tag{29}$$

and reads

$$M_h^2 \leq \lambda_5 v_o^2. \tag{30}$$

It can be easily shown that for electroweak breaking driven only by doublets (so that $\rho = 1$ automatically) all the bounds are exactly the same. In that case $v_e = 0$ and $\phi_1 = \phi_5$ (implying $\lambda_1 = \lambda_5$) and the bounds (19) and (30) coincide trivially. To see that also the other three bounds are the same, notice that a suitable rotation in field space will permit us to write the doublet responsible of electroweak breaking as

$$\Phi = \left(\begin{array}{c} G^+ \\ \frac{1}{\sqrt{2}}(v_o + \phi_1) + \frac{i}{\sqrt{2}}G^0 \end{array}\right), \tag{31}$$

while the rest of doublets will play no role. Then the quartic coupling for Φ is written as

$$\delta V = \frac{1}{2}\lambda(\Phi^\dagger\Phi)^2, \tag{32}$$

and expanding in components one gets[3]

$$\lambda = \frac{1}{3}\lambda_2 = \lambda_3 = \frac{1}{2}\lambda_4. \tag{33}$$

Inserting these relations in the expressions (25-27) we recover always the result λv_o^2.

In the most general case, with multiplets of different kinds contributing to the breaking, the bounds will be different and one has to choose the stronger. In addition, for particular models with extra symmetries, further bounds can be derived which may compete with the ones given here.

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[3]Actually this holds whenever the Higgs potential has a custodial $SU(2)_{L+R}$ symmetry (to ensure $\rho = 1$).

4. The decoupling limit

From the mass bounds written in the form (15) some extra useful information can be extracted. Consider one particular bound with $\phi_i = \varphi_i \equiv \phi$ so that (15) reads

$$\sum_A M_A^2 |\langle\phi|A\rangle|^2 \leq \lambda v^2. \tag{34}$$

As noted in refs. [1, 2] large masses can enter the sum (34) and then the corresponding eigenstates will have a small overlapping with ϕ:

$$|\langle\phi|A\rangle|^2 \leq \lambda \frac{v^2}{M_A^2}. \tag{35}$$

When all scalars but one are much heavier than the electroweak scale ($M_H \gg v$), eq. (34) tells that the light state $|1\rangle$ is predominantly [4] $|\phi\rangle$:

$$|\langle\phi|1\rangle|^2 = 1 - \mathcal{O}(v^2/M_H^2). \tag{36}$$

This in turn determines the properties of the light scalar in this decoupling limit. This issue has been studied in the literature concentrating mainly in the two-doublet model [9], where one finds that the light Higgs has standard couplings making extremely difficult to unravel the non minimal structure of the Higgs sector.

Here we will concentrate on a different aspect of this problem. Suppose that v_e is non-zero (*e.g.* some non-doublets contribute to electroweak breaking) so that the bounds *1* and *5* in the previous section are different. In such a case, what is the composition of the light Higgs in the decoupling limit? The first bound will give $|1\rangle = |\phi_1\rangle$ while the fifth will rather say $|1\rangle = |\phi_5\rangle$.

The way out of this paradox is that the decoupling limit cannot be reached in that case, that is, one cannot arrange the model so that all the scalars but one are heavy. In other words, the combination of two different bounds, associated with two different field space directions, should provide a bound on the second-to-lightest scalar particle. This bound can be easily derived. Let us call $M_{1,2}^2$ the squared masses of the two light scalars ($M_2^2 > M_1^2$) and α_{15} the angle ($0 < \alpha_{15} \leq \pi/2$) between the two field directions ϕ_1 and ϕ_5. Next, $P|1\rangle$ is the (normalized) projection of the lightest scalar eigenstate onto the plane spanned by ϕ_1 and ϕ_5. Call β the angle ($-\pi/2 \leq \beta \leq \pi/2$) between $P|1\rangle$ and $|\phi_1\rangle$. Examining the quantities $\langle\phi_1|M^2 - M_1^2|\phi_1\rangle$ and $\langle\phi_5|M^2 - M_1^2|\phi_5\rangle$ we get the inequalities:

$$\begin{aligned}
(M_2^2 - M_1^2)\left[1 - |\langle\phi_1|1\rangle|^2\right] &\leq \lambda_1 v_1^2 - M_1^2, \\
(M_2^2 - M_1^2)\left[1 - |\langle\phi_5|1\rangle|^2\right] &\leq \lambda_5 v_5^2 - M_1^2.
\end{aligned} \tag{37}$$

Noting that

$$\begin{aligned}
|\langle\phi_1|1\rangle|^2 &\leq |\langle\phi_1|P|1\rangle|^2 = \cos^2\beta, \\
|\langle\phi_5|1\rangle|^2 &\leq |\langle\phi_5|P|1\rangle|^2 = \cos^2(\alpha_{15} - \beta),
\end{aligned} \tag{38}$$

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[4] The mass of this light Higgs is usually maximized in this limit. Note however that, even if $\langle\phi|M^2|\phi\rangle = \lambda v^2$, the mass will not saturate the bound λv^2 necessarily.

it then follows

$$
\begin{aligned}
M_2^2 - M_1^2 \;\; &\leq \;\; \max_{|1\rangle} \left\{ \min \left[\frac{\lambda_1 v_1^2 - M_1^2}{1 - |\langle \phi_1 | 1 \rangle|^2}, \frac{\lambda_5 v_5^2 - M_1^2}{1 - |\langle \phi_5 | 1 \rangle|^2} \right] \right\} \\
&\leq \;\; \max_{-\pi/2 \leq \beta \leq \pi/2} \left\{ \min \left[\frac{\lambda_1 v_1^2 - M_1^2}{\sin^2(\alpha_{15} - \beta)}, \frac{\lambda_5 v_5^2 - M_1^2}{\sin^2 \beta} \right] \right\} \qquad (39) \\
&= \;\; \frac{1}{\sin^2 \alpha_{15}} \left\{ \lambda_1 v_1^2 + \lambda_5 v_5^2 - 2M_1^2 + 2 \left[(\lambda_1 v_1^2 - M_1^2)(\lambda_5 v_5^2 - M_1^2) \cos^2 \alpha_{15} \right]^{1/2} \right\}.
\end{aligned}
$$

Or going even further

$$
M_2^2 - M_1^2 \;\; \leq \;\; \frac{1}{\sin^2 \alpha_{15}} \left\{ \left[\lambda_1 v_1^2 - M_1^2 \right]^{1/2} + \left[\lambda_5 v_5^2 - M_1^2 \right]^{1/2} \right\}^2 \sim \frac{\lambda v^2}{\sin^2 \alpha}.
$$

As is clear from this last expression the bound disappears for $\sin \alpha \to 0$ which corresponds to the situation of bounds with equal field directions (or, in the particular case we analyzed, to $v_e \to 0$). In practice, for $\sin^2 \alpha \sim \lambda v^2 / M_H^2$ the mass of the second-to-lightest Higgs can (in principle) be as heavy as M_H and the decoupling limit with only one light Higgs can be realized. Note that, in the derivation of (39) the presence of α_{15} in the denominator $\sin^2(\alpha_{15} - \beta)$, is crucial to avoid the possibility of both denominators going to zero simultaneously in which case the bound would be lost.

It is clear that one can combine any two independent bounds in the same way as we have done and obtain different bounds on M_2^2. By independent bounds we mean bounds with linearly independent associated field directions[5]. It should be clear then that from three independent bounds a limit on the mass of the third light scalar can be extracted. In general, having N bounds $\mathcal{B}_a$, with associated field directions $\varphi_a = \phi_a$, the masses of M lighter scalars can be bounded, with $M = rank\{\phi_a\} \leq N$.

5. Production Cross Sections

Besides putting limits on the scalar masses it is important to know the composition of the light states in order to see if they can be produced at all in accelerators. The paradigmatic situation is exemplified by the next to minimal Supersymmetric Standard Model, NMSSM, which contains an extra chiral singlet in addition to the MSSM particle content. As is well known, a light scalar should be present in the spectrum of this model, but it can be singlet dominated and then hard to produce. In an interesting paper, ref. [10], it was shown that in the circumstance of the lightest Higgs boson being predominantly a singlet the second to lightest Higgs scalar will have an upper mass bound not far from the original bound on the lightest Higgs mass. And in case that also this second Higgs is singlet dominated, the

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[5]For example, bounds 2, 3, 4 are not linearly independent. Moreover, it can be shown that, if two independent bounds $\langle \phi_a | M^2 | \phi_a \rangle \leq \lambda_a v_a^2$ and $\langle \phi_b | M^2 | \phi_b \rangle \leq \lambda_b v_b^2$ exist, the off-diagonal matrix element satisfies $\langle \phi_a | M^2 | \phi_b \rangle \leq \sqrt{\lambda_a \lambda_b} v_a v_b$. For this reason, in this section we have restricted the discussion to diagonal bounds. Off-diagonal bounds, like bound 3 of the previous section are spurious.

third scalar will be subject to a similar bound. This led to the conclusion that one of the three scalars will be produced in a future e^+e^- linear collider (operating at $\sqrt{s} \sim 300\ GeV$) abundantly enough to guarantee detection. In this section we will show that such ladder of upper bounds can be generalized and applies to all models we are considering. The detectability at a Next Linear Collider can be studied in particular cases using these results and following the same procedure of Ref. [10] (see also [11]).

To derive the bounds we use the field ϕ_3 of section 3. The superposition of a scalar eigenstate H_i^0 with ϕ_3 is a measure of the strength of the gauge coupling of that eigenstate:

$$\langle \phi_3 | H_i^0 \rangle \sim \frac{ZZH_i^0}{ZZH_{SM}}. \tag{40}$$

Then consider the quantities $\langle \phi_3 | M^2 - m_N^2 | \phi_3 \rangle$, where m_N^2 is the squared mass of the Nth scalar eigenstate. For $N = 1$ we reproduce the bound of section 3:

$$m_1^2 \leq \lambda_3 v_3^2. \tag{41}$$

For N=2 is easy to get the inequality

$$m_2^2 \leq \frac{\lambda_3 v_3^2 - V_{11}^2 m_1^2}{1 - V_{11}^2}, \tag{42}$$

where $V_{11}^2 = |\langle H_1^0 | \phi_3 \rangle|^2$. In general one obtains for the Nth eigenstate

$$m_N^2 \leq \frac{\lambda_3 v_3^2 - \Sigma_N^2 m_1^2}{1 - \Sigma_N^2}, \tag{43}$$

with

$$\Sigma_N^2 = \sum_{p=1}^{N-1} V_{1p}^2 = \sum_{p=1}^{N-1} |\langle H_p^0 | \phi_3 \rangle|^2. \tag{44}$$

As explained in ref. [10] the limit of small Σ_N corresponds to the case of the first $N-1$ scalars being mostly decoupled from the Z boson. In that case the bound on the Nth scalar is stronger.

From these mass bounds, and knowing the couplings to the Z boson one can put lower bounds on the production cross sections and study the capabilities of future e^+e^- linear colliders for the discovery of one of the neutral scalars of a given model.

7. Conclusions

We have presented a novel and simple bound on the (tree-level) mass of the lightest neutral scalar in multi Higgs electroweak models. Whenever the Higgs sector is weakly coupled the bound presented implies the existence of a scalar state at or below the scale of symmetry breaking. This result has applications in many different models of electroweak symmetry breaking and can be generalized to any model in which a continuous symmetry group is spontaneously broken. Here we have restricted our attention, as an example, to

general low-energy supersymmetric models. We have reproduced some previous analytical bounds in a straightforward manner and clarified the limits of applicability of the usual method to extract the bound in such models.

Also, by comparing our new bound with previous general bounds existent in the literature, we have shown how to extract in some cases extra information on the mass of heavier neutral scalar states. Some implications for the so-called decoupling limit in multi Higgs models have been obtained.

Finally we have generalized previous interesting results of Kamoshita et al. (obtained for the NMSSM, ref. [10]) concerning the detectability of light neutral Higgses at the NLC.

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Appendix

We present here an example of supersymmetric model for which the usual technique to extract an "electroweak-controlled" bound on the mass of the lightest Higgs boson (see e.g. Ref. [5]) would fail. The field content of the model is that of the MSSM supplemented by a chiral $SU(2)_L$ triplet $\hat{\Sigma}$, with hypercharge 1, and a 4-plet $\hat{\Xi}$ with $Y = -1/2$, plus all extra fields necessary to cancel anomalies. The Yukawa couplings and VEVs of these extra fields will be assumed to be negligible so that we will ignore them in the following, its presence being unimportant for the effect we want to discuss.

Let us write the fields Σ_{ij} and Ξ_{ijk} $(i, j, k = 1, 2)$ in the following matrix representation

$$\Sigma_{ij} = \begin{pmatrix} \sigma_2^+/\sqrt{2} & -\sigma_1^{++} \\ \sigma_3^0 & -\sigma_2^+/\sqrt{2} \end{pmatrix},$$

$$\Xi_{1ij} = \begin{pmatrix} -\xi_2^0/\sqrt{3} & \xi_1^+ \\ -\xi_3^-/\sqrt{3} & \xi_2^0/\sqrt{3} \end{pmatrix} \quad , \quad \Xi_{2ij} = \begin{pmatrix} -\xi_3^-/\sqrt{3} & \xi_2^0/\sqrt{3} \\ -\xi_4^{--} & \xi_3^-/\sqrt{3} \end{pmatrix}. \tag{45}$$

The superpotential contains a part

$$\delta f = \epsilon_{ij} \left(\mu H_{1i} H_{2j} + \lambda H_{1i} \Sigma_{jk} H_{1k} + \sqrt{3}\zeta H_{1i} \Xi_{jkl} \Sigma_{lk} \right) + 3\gamma \Sigma_{ij} \Xi_{1jk} \Xi_{2ki}, \tag{46}$$

and for the neutral components we get

$$f = \mu h_1^0 h_2^0 + \lambda \sigma_3^0 h_1^0 h_1^0 + \zeta h_1^0 \sigma_3^0 \xi_2^0 + \gamma \xi_2^0 \xi_2^0 \sigma_3^0 \tag{47}$$

from which we can obtain the potential for $h_1^0, h_2^0, \sigma_3^0, \xi_2^0$. In general these fields will develop VEVs v_1, v_2, s_3, x_2 and, to satisfy the experimental constraints on $\Delta\rho$, one has to impose either that $s_3^2, x_2^2 \ll v_1^2 + v_2^2$ or $s_3^2 \simeq 3x_2^2$.

In any case, it is straightforward to obtain the mass matrix for the real fields $h_1^{0r}, h_2^{0r}, \sigma_3^{0r}, \xi_2^{0r}$ in the vacuum (v_1, v_2, s_3, x_2). Looking only to the h_1^{0r}, h_2^{0r} submatrix one gets (neglecting gauge contributions for simplicity):

$$
\begin{aligned}
M_{11}^2 &= -(B + 2\lambda s_3)\mu \tan\beta - \zeta A s_3 \frac{x_2}{v_1} \\
&+ 4\lambda^2 v_1^2 + 3\lambda\zeta v_1 x_2 - \zeta\gamma\frac{x_2^3}{v_1} - 2(\lambda + \gamma)\zeta s_3^2\frac{x_2}{v_1}, \\
M_{12}^2 &= (B + 2\lambda s_3)\mu, \\
M_{22}^2 &= -(B + 2\lambda s_3)\mu \cot\beta - \zeta\mu s_3\frac{x_2}{v_2},
\end{aligned}
\tag{48}
$$

where B, A are soft masses. We see that this matrix is not of the general form obtained in [5] although the model satisfies all the conditions required in that paper. The expressions given above show clearly that the bound one can obtain from this 2×2 matrix is not controlled by the electroweak scale.

The situation resembles that encountered (see [7]) in the Supersymmetric Singlet Majoron Model [8] and it has a similar solution [6]. Let us redefine the fields[6]

$$
\begin{aligned}
\phi_1 &= h_1^{0r}, \\
\phi_2 &= \frac{1}{v_2'}(v_2 h_2^{0r} + x_2 \xi_2^{0r}),
\end{aligned}
\tag{49}
$$

where $v_2'^2 = v_2^2 + x_2^2$. The mass submatrix for ϕ_1, ϕ_2 has now the well-behaved form

$$
M'^2 = \left\| \begin{array}{cc} -m_3'^2 \tan\beta' + \Delta_{11} & m_3'^2 + \Delta_{12} \\ m_3'^2 + \Delta_{12} & -m_3'^2 \cot\beta' + \Delta_{22} \end{array} \right\|,
\tag{50}
$$

with

$$
m_3'^2 = (B + 2\lambda s_3)\frac{v_2}{v_2'} + \zeta A s_3 \frac{x_2}{v_2'} \quad , \quad \tan\beta' = \frac{v_2'}{v_1},
\tag{51}
$$

and

$$
\begin{aligned}
\Delta_{11} &= 4\lambda^2 v_1^2 + \frac{x_2}{v_1}\left[3\lambda\zeta v_1^2 - \zeta\gamma x_2^2 - 2(\lambda + \gamma)\zeta s_3^2 \right], \\
\Delta_{12} &= \frac{x_2}{v_2'}\left[2(\zeta^2 + 2\gamma\lambda)v_1 x_2 + \zeta\lambda(3v_1^2 + 2s_3^2) + \zeta\gamma(3x_2^2 + 2s_3^2) \right], \\
\Delta_{22} &= \frac{x_2}{v_2'^2}\left[4\gamma^2 x_2^3 + \zeta\gamma(3x_2^2 - 2s_3^2)v_1 - \zeta\lambda(2s_3^2 + v_1^2)v_1 \right].
\end{aligned}
\tag{52}
$$

From this matrix the Yukawa-part of the mass bound can be obtained as

$$
\delta m_h^2 = \Delta_{11}\cos^2\beta' + \Delta_{22}\sin^2\beta' + \Delta_{12}\sin 2\beta' = 4(\lambda c_1^2 + \zeta c_1 c_x + \gamma c_x^2)^2 v'^2,
\tag{53}
$$

---

[6]Note that ξ_2^{0r} and h_2^{0r} belong to different $SU(2)_L$ representations so that (49) is not a mere redefinition of the "correct" doublet fields.

with $v'^2 = v_1^2 + v_2^2 + x_2^2$, $c_i = v_i/v'$, $c_x = x_2/v'$. Of course, this is the result one can obtain using the simple prescription given in the text. Defining $\phi^0 = c_1 h_1^0 + c_2 h_2^0 + c_x \xi_2^0$ and recasting the superpotential (47) in the form

$$f = (\lambda c_1^2 + \zeta c_1 c_x + \gamma c_x^2) \sigma_3^0 \phi^0 \phi^0 + ... \tag{54}$$

gives directly the bound (53).

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